

Maths Pack

Distance Learning – Mathematics Support Pack

For the University Certificates in Astronomy and Cosmology

These certificate courses are for your enjoyment. However, a proper study of astronomy or cosmology at University level also requires some elements of both physics and mathematics. The appropriate use of numbers enables you to appreciate the extreme scales and sizes that we encounter whilst describing the universe. Using **numbers** rather than everyday adjectives like “big” or “small” allows us to be much more quantitative and to contrast scales properly. Even so, the sheer sizes may still be difficult to grasp, so don't be put off when the numbers are very big or very small.

If you are uneasy about mathematics you should work through the first half of the maths pack to gain confidence before starting your course. In some areas of the course you will meet some arithmetic and equations. **Arithmetic** is usually performed using a scientific calculator and involves operations such as addition and multiplication and is no more complex than that found in senior school.

We have also included **equations** in the courses. In many cases you could omit the equations, concentrating instead on other aspects of your course. You would acquire a good understanding of the concepts involved and probably still obtain a respectable pass mark. However we strongly recommend that you have a go with the equations and once you become familiar with the symbols, you may well be surprised how straightforward the mathematics is. But you should be aware that to achieve high marks in this course or to pursue your studies in astronomy at a higher level, you would definitely need to master equations. Our certificates provide a taster for how the subject material develops beyond the level of your current course.

This Maths Pack is intended to help boost your confidence with the maths you will meet in the Certificate Courses. Study the examples in each section then try the similar questions, comparing your answers with those at the end of the Maths Pack. If you require additional help remember there is a **telephone tutorial support line** as well as the course web site. Both are described elsewhere in the course information.

Powers of ten

Astronomy makes use of very large *and* very small numbers. They are usually expressed as **powers of 10**. “Ten to the power of two” is 10^2 (or 10×10). “Ten to the power of three” is 10^3 (or $10 \times 10 \times 10$). Below you will see some examples of different powers of ten. Complete the missing entries below and also **make sure you can key all these numbers into your pocket calculator**.

$$10^6 = 1,000,000$$

$$10^4 = 10,000$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^7 =$$

$$10^2 =$$

$$10^5 =$$

$$10^{-1} = 0.1$$

$$10^{-3} = 0.001$$

$$10^{-4} =$$

$$10^{-6} =$$

$$10^{-2} =$$

Check that you can also work backwards, converting numbers that show all the digits, into numbers represented as powers of ten.

$$100 = 10^2$$

$$1 = 10^0$$

$$100,000 =$$

$$10 =$$

$$0.001 =$$

$$0.0001 =$$

$$10,000,000 =$$

Multiplying powers of 10

Multiplying ordinary numbers such as 2×2 is straightforward, but when powers of ten are involved we need to look carefully at how the powers are combined. Study the examples below and then fill in the blanks in the remaining examples. Take care with the minus sign.

$$10^6 \times 10^3 = 1,000,000 \times 1,000 = 10^{6+3} \\ = 1,000,000,000 = 10^9$$

$$10^4 \times 10^4 = 10^{4+4} = 10^8$$

$$10^2 \times 10 =$$

$$10^3 \times 10^{-2} = 10^{3-2} = 10$$

$$10^6 \times 10^7 =$$

$$10^{-2} \times 10^4 =$$

$$10^{26} \times 10^3 =$$

$$10^{-34} \times 10^9 =$$

The previous method for multiplying two powers of ten may be extended to cover products of several powers of ten:

$$10^3 \times 10^4 \times 10^3 = 10^{3+4+3} = 10^{10}$$

$$10 \times 10^2 \times 10^4 = 10^{1+2+4} = 10^7$$

$$10^4 \times 10^7 \times 10^6 =$$

$$10^2 \times 10^{26} \times 10^{-12} =$$

$$10^7 \times 10^{-4} \times 10^3 =$$

$$10^{12} \times 10^{-4} \times 10^3 =$$

$$10^{-34} \times 10^{24} \times 10^5 =$$

Dividing by powers of ten

When we divide by a power of ten, it is the same as multiplying by the power of ten with the *opposite* sign. So in the following example we divide 1 by a large number resulting in a small number:

$$\frac{1}{10^3} = 1 \times 10^{-3} = 10^{-3}$$

Conversely if we divide by a small number the result is a large number:

$$\frac{1}{10^{-4}} = 10^4$$

This idea reappears in the following examples:

$$\frac{10^6 \times 10^6}{10^3} = 10^6 \times 10^6 \times 10^{-3} = 10^{6+6-3} = 10^9$$

$$\frac{10}{10^2 \times 10^{14}} = 10^{1-2-14} = 10^{-15}$$

$$\frac{10 \times 10^{-2}}{10^{-3}} = 10^{1-2+3} = 10^2$$

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Complete the following examples **without** using a calculator. Then check your results using a calculator.

$$\frac{10^7 \times 10^9 \times 10^{-3}}{10^2} =$$
$$\frac{10^{-2} \times 10^4 \times 10^6}{10^{-3} \times 10^{-2}} =$$
$$\frac{1}{10^2 \times 10^5 \times 10^{-3}} =$$
$$\frac{10^9}{10^2 \times 10^{-34} \times 10^{-27}} =$$
$$\frac{10^{16} \times 10^{-9} \times 10^4}{10^{-14}} =$$

Summary of rules for combining powers of 10

$$10^x \times 10^y = 10^{x+y}$$

$$10^x \div 10^y = 10^{x-y}$$

$$(10^x)^y = 10^{x \times y} = 10^{xy}$$

For explanation of this last rule, see under **Powers in general**.

Scientific notation

Some numbers are not just simple powers of ten. Here are a few examples.

$$2,000 = 2 \times 10^3$$

$$3,000,000 = 3 \times 10^6$$

$$1,600,500 = 1.6005 \times 10^6$$

Many of the numbers you see in your course will be expressed as the product of a number between 1 and 10 and a power of ten. This is often called **scientific notation**.

Practise entering each of the numbers above into your scientific calculator.

When you perform a calculation involving a mixture of numbers it is normal to express the result in scientific notation, especially if the result is very large or very small.

$$2^2 \times 10^2 \times 1.5 \times 10 = 6 \times 10^3$$

$$4 \times 100 \times 6.6 \times 10^{-2} =$$

$$7 \times 10 \times 3.14 \times 2 \times 10^3 =$$

Powers in general

It is possible to raise *any* number to the power of something. It does not need to be a power of ten as the next example shows.

$$10^2 = 10 \times 10 = 100$$

Now let us replace the “10” by another number say “2”.

$$2^2 = 2 \times 2 = 4$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$2^3 =$$

$$2^6 =$$

We would read this last example out loud as “two raised to the power six”. When the power is large you may find it useful to resort to a calculator to evaluate the result.

Now evaluate the following in the same way:

$$3^2 =$$

$$5^4 =$$

Here is an example of how to calculate powers when the number in question is a power of 10. We have seen that a number raised to the power of 2 means “multiply that number by itself 2 times. So:

$$(10^3)^2 = (10^3) \times (10^3) = 10^6$$

$$(10^2)^4 = (10^2) \times (10^2) \times (10^2) \times (10^2)$$
$$= 10^{(4 \times 2)} = 10^8$$

This is summarised by the third of the rules in the summary of **Powers of ten**.

Evaluate the following:

$$(10^4)^3 =$$

$$(10^5)^4 =$$

Square roots

Consider the following examples:

$$2^2 = 2 \times 2 = 4$$

$$3^2 = 3 \times 3 = 9$$

$$4^2 = 4 \times 4 = 16$$

What number when raised to the power of 2 gives the number 16? The answer (4 in this case) is the **square root** of the number in question. This is an important concept and there is a special symbol, $\sqrt{\quad}$, for this.

$$\sqrt{16} = \sqrt{4 \times 4} = 4$$

$$\sqrt{9} = \sqrt{3 \times 3} = 3$$

$$\sqrt{49} =$$

$$\sqrt{36} =$$

$$\sqrt{20} =$$

Expressions involving the square root can occur in equations. See for example radius and area of a circle in **Rearranging equations**.

Logarithms

Some physical processes such as the eye's response are by their very nature logarithmic. So the equations defining magnitudes contain logarithms. Logarithms can arise elsewhere in the maths or they can just be used as a convenient way of displaying a large range of numbers on the same plot.

For a number that is a simple power of ten, its logarithm (or log) is just the power of ten.

$$\log(100) = \log(10^2) = 2$$

$$\log(10000) = \log(10^4) = 4$$

$$\log\left(\frac{1}{10}\right) = \log(10^{-1}) = -1$$

$$\log(10^{-2}) = -2$$

$$\log(10^5) = 5$$

In simple cases like this we can work out the result in our head. More complicated numbers require the use of a scientific calculator. Most calculators have two types of logs. (Usually the keys have labels **ln** and **log**). All of our examples use **logs to the base of ten**, which

can be found by using the “**log**” key on your calculator.

Now complete the following examples

- without using a calculator
- using a calculator.

$$\log(1000) = 3$$

$$\log(10^4) =$$

$$\log(10^7) =$$

$$\log(10^5) =$$

$$\log(10^{-2}) =$$

$$\log(10^{-8}) =$$

The logarithms of numbers that are not simple powers of ten can be found using a pocket calculator or (log tables!)

$$\log 2 = 0.301$$

$$\log 3 = 0.4771$$

$$\log(1.2) = 0.0792$$

$$\log(12) = 1.0792$$

$$\log(120) = 2.0792$$

$$\log(1200) = \log(1.2 \times 10^3) = 3.0792$$

Try the following using a calculator:

$$\log 5 =$$

$$\log(2010) =$$

$$\log(0.45) =$$

$$\log(75000) =$$

If we need to find the logarithm of an expression, this is usually written in brackets, and we must work this out first.

$$\log(5 + 6) = \log 11 = 1.041$$

$$\log(2 \times 10) =$$

$$\log \frac{4}{2} = \log \left(\frac{4}{2} \right)$$

$$\log \frac{9}{3} =$$

In equations like the following, we must **obtain the value of the expression in brackets before finding its log and proceeding with the other arithmetic**.

$$2.5 \log\left(\frac{9}{3}\right) = 2.5 \times \log 3 = 2.5 \times 0.477$$

$$= 1.1928$$

$$2.5 \log\left(\frac{3 \times 10^6}{100}\right) = 2.5 \times \log(30,000)$$

$$= 2.5 \times 4.4771 = 11.1928$$

Try the following:

$$2.5 \log 14 = 2.5 \times \log 14 =$$

$$3.5 \log\left(\frac{21}{3}\right) =$$

$$7 \log\left(\frac{21 \times 2}{3}\right) =$$

Inverse logarithms

What if we had to find the number, x , that had a log of 5? This is the inverse of taking logs. The result is ten to the power of 5.

$$\log x = 5$$

Note that the function name appears in normal text whereas the variable, x , (representing an unknown number) appears in italic print.

$$x = 10^5$$

Most scientific calculators will have a key for inverse logs. It is usually the same key as the log but accessed through a “function key”. Use a calculator to find x in each of the following:

$$\log x = 4.5 \quad x = 10^{4.5} =$$

$$\log x = 3.2 \quad x =$$

$$\log x = 6.5 \quad x =$$

$$\log x = -2 \quad x =$$

Order of evaluation

The order of evaluation is important. In school this is often called BODMAS. (**B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction. This rule tells us that we must work through the equations first doing the bits in brackets, then any functions (like logarithms) then division or multiplication and lastly any addition or subtraction.

Each step is identified in the following example.

- First evaluate the expression for which we are going to find the logarithm. Here it is also written in brackets as $\left(\frac{15}{3}\right)$ for clarity, but the brackets are optional.
- Then take the **log** of this result before multiplying it by (-2.5).
- Lastly carry out the addition.

$$4 - 2.5 \log\left(\frac{15}{3}\right) = 4 - 2.5 \times \log 5$$

$$= 4 - 2.5 \times 0.6990$$

$$= 4 - 1.7470 = 2.2526$$

In most cases the equations are expressed in a way which makes the order of evaluation fairly obvious.

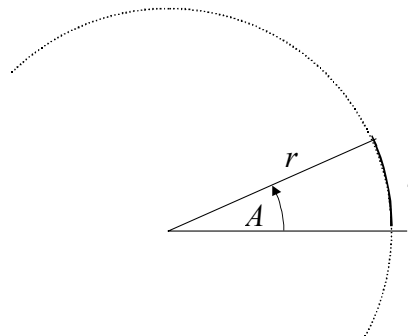
$$5 - 3 + 2.5 \log \frac{20}{5} =$$

$$6 + 4 \log \frac{10^5}{2} =$$

$$45 + \log 34 =$$

Degrees and radians

The angular separation between two lines can be measured in degrees or radians. The following diagram shows two intersecting straight lines. The arc, of length l measures the opening at a distance r from the point of intersection.



The angle A , measured in radians, is related to r and l through the equation:

$$A = \frac{l}{r}$$

There are 2π radians in a full circle.

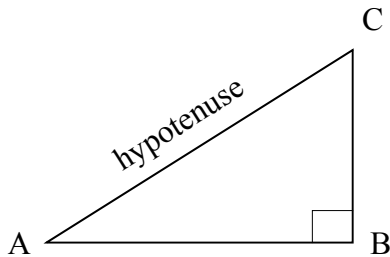
Depending on the type of problem that we are working on we may want express an angle in degrees or radians. As there are 360° in a full circle it is easy to convert from degrees to radians and *vice versa*.

$$\text{radians} = \frac{2 \times \pi}{360} \times \text{degrees}$$

$$\text{degrees} = \frac{360}{2 \times \pi} \times \text{radians}$$

Trigonometric functions

Some areas of astronomy make use of standard trigonometric functions: **sin** (sine), **cos** (cosine) and **tan** (tangent). All three functions are derived from the sides of a right angle triangle, which in the equations below are represented by AB, BC and AC.



$$\sin A = \frac{BC}{AC}$$

$$\cos A = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$

The angle A can be measured in either degrees or radians. Most calculators have a switch to toggle between the two units.

Use a calculator to find the sine of the following angles:

$$\sin 30^\circ = 0.500$$

$$\sin 60^\circ = 0.8660 \quad \left(= \frac{\sqrt{3}}{2} \right)$$

$$\sin 45^\circ =$$

$$\sin 90^\circ =$$

$$\sin 6^\circ =$$

Use your calculator to find the following:

$$\sin 0^\circ =$$

$$\cos 0^\circ =$$

$$\tan 0^\circ =$$

$$\sin 90^\circ =$$

$$\cos 90^\circ =$$

$$\tan 45^\circ =$$

Inverse trig functions

Sometimes we know the value of the sine, but not the angle A . Here is an example:

$$\sin A = 0.45$$

To find the angle we use the inverse sine function \sin^{-1} . Your calculator will have a special key to do this operation. The value A in the above example is found from:

$$A = \sin^{-1} 0.45 = 26.744^\circ$$

You could read this to yourself as “ A is the angle whose sine is 0.45.” This might help to overcome a common misunderstanding

$\sin^{-1} 0.45$ does **not** mean

$$A = \frac{1}{\sin 0.45}$$

To get this expression you would write:

$$A = (\sin 0.45)^{-1} = \frac{1}{\sin 0.45}$$

If we know the lengths of the appropriate sides BC and AC in the right-angled triangle we can calculate the sine of the angle A and hence the angle itself. The following assumes that the angle A is in degrees.

Given info		Derived info		Result
Side BC	Side AC	BC/AC	Sin A	A
4	5	4/5	0.8	53.13
3.5	6	3.5/6	0.583	35.69

Now try the following examples:

Side BC	Side AC	Sin A	A
5	6		
2	5		
1.5	4		

Using symbols in equations

Because of the size of many numbers it is convenient to write equations using symbols. Here is a simple example for the area of a circle.

$$\text{Area} = \pi r^2$$

In this equation π (‘pi’) is the number with approximate value 3.1416 and r is the radius of a circle. If r is 3 then:

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$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= 3.1416 \times r \times r \\ &= 3.1416 \times 3 \times 3 \\ &= 28.274 \end{aligned}$$

The area may be given the symbol A so that the equation can be written:

$$A = \pi r^2$$

Now find the area of a 10p coin (in units of square millimetres). Measure the diameter in millimetres using a ruler and then calculate the radius. Now use the above equation to calculate the area.

Suppose you are presented with an equation of the form:

$$L = 4\pi r^2 \sigma T^4$$

There are two powers in this equation which could be written out in long hand:

$$L = 4\pi \times (r \times r) \times \sigma \times (T \times T \times T \times T)$$

To calculate L (the answer) all we have to do is make sure that we know the numerical value for each of the symbols to the right of the equal sign.

Calculate L when $T = 6000$, $r = 6 \times 10^8$ and $\sigma = 5.67 \times 10^{-8}$.

$$\begin{aligned} L &= 4\pi r^2 \sigma T^4 \\ &= 4 \times 3.1416 \times (6 \times 10^8)^2 \times 5.67 \times 10^{-8} \times 6000^4 \\ &= 12.566 \times 3.6 \times 10^{17} \times 5.67 \times 10^{-8} \times 1.296 \times 10^{15} \\ &= 12.566 \times 3.6 \times 5.67 \times 1.296 \times 10^{17-8+15} \\ &= 332.4 \times 10^{24} \\ &\approx 3.3 \times 10^{26} \end{aligned}$$

Another straightforward example is based on the calculation of the total mass in a sphere. Let M represent the mass, r the radius of the sphere and ρ the density of material.

$$M = \text{Volume} \times \text{density} = \frac{4\pi r^3}{3} \rho$$

The values to use are:

$$\begin{aligned} r &= 7 \times 10^8 \text{ m} \\ \rho &= 1400 \text{ kg m}^{-3} \\ \pi &= 3.1416 \end{aligned}$$

Hence the mass is calculated from:

$$\begin{aligned} M &= \frac{4 \times 3.1416 \times (7 \times 10^8)^3}{3} \times 1400 \\ &= 2.011 \times 10^{30} \approx 2 \times 10^{30} \text{ kg} \end{aligned}$$

Some of the equations in the certificate courses contain expressions that are a little more complicated. The following equation describes the magnitudes of two stars:

$$m_1 - m_2 = -2.5 \log \left(\frac{f_1}{f_2} \right)$$

m_1 , m_2 , f_1 and f_2 each represents a number. The 1 and 2 are called subscripts. Here they are used to show which quantity refers to which star. Questions will often specify the values of all but one variable leaving you to calculate the missing value.

Suppose that $f_1 = 120$ and $f_2 = 30$ so that the right hand side of the equal sign can be calculated. The variable m_2 may also be specified. Let's suppose $m_2 = 3$.

$$m_1 - m_2 = -2.5 \log \left(\frac{f_1}{f_2} \right)$$

$$m_1 - 3 = -2.5 \log \left(\frac{120}{30} \right)$$

$$\begin{aligned} m_1 &= 3 - 2.5 \log 4 \\ &= 3 - 2.5 \times 0.602 \\ &= 1.4948 \end{aligned}$$

Note that in the third line above, the unknown m_1 was obtained on its own on the left-hand side of the equation by adding 3 to both sides. This follows the rules described in the next section about rearranging equations.

Solve the same equation above for m_1 for the four different sets of m_2 , f_1 and f_2 given in this table.

	m_2	f_1	f_2	Answer
1	6	65	40	
2	4	350	12	
3	5	137	4	
4	8	14	30	

Rearranging equations

Sometimes to perform a calculation we will have to rearrange an equation to isolate the variable that we need.

All equations are presented with an expression on both sides of the equal sign. To rearrange the equation we make identical operations on each expression. (Make sure you change the entire expression and not just a small part of it.)

For example, consider the area of a circle. Suppose that the area is known and we wish to derive the radius. This means that it would be better to write the equation with the radius alone on the left-hand side of the equal sign. The standard way of presenting the equation is as follows:

$$A = \pi r^2$$

You are allowed to add, subtract divide or multiply the equation by anything provided that you do exactly the same to both sides. The first step is to divide both sides by pi.

$$\begin{aligned} \frac{A}{\pi} &= \frac{\pi r^2}{\pi} \\ \Rightarrow \frac{A}{\pi} &= r^2 \end{aligned}$$

Now we can take the square root of both sides

$$\begin{aligned} \sqrt{\frac{A}{\pi}} &= \sqrt{r^2} \\ \Rightarrow \sqrt{\frac{A}{\pi}} &= r \end{aligned}$$

Thus the radius can be calculated from the area:

$$r = \sqrt{\frac{A}{\pi}}$$

The same principles can be applied to other equations that we have seen. In the example above on stellar magnitude, suppose we had needed to find f_2 . This looks difficult since the variable is buried deep on the right hand side of the equation.

$$m_1 - m_2 = -2.5 \log\left(\frac{f_1}{f_2}\right)$$

This will require the rearrangement of the equation, so that the unknown f_2 appears on the left.

$$(m_1 - m_2) = (-2.5) \times \log\left(\frac{f_1}{f_2}\right)$$

$$\frac{(m_1 - m_2)}{(-2.5)} = \log\left(\frac{f_1}{f_2}\right)$$

We now need to take antilogs:

$$10^{\frac{(m_1 - m_2)}{-2.5}} = \frac{f_1}{f_2}$$

So we take the inverse (or turn both sides upside down – which includes changing the sign of the power on the right hand side.)

$$\frac{f_2}{f_1} = 10^{\left(\frac{m_1 - m_2}{2.5}\right)}$$

$$f_2 = f_1 10^{\left(\frac{m_1 - m_2}{2.5}\right)}$$

Now try the following, rearranging each equation so that the variable given is alone on the left-hand side.

$$y = m x + c \quad \text{in terms of } m$$

$$L = 4\pi r^2 \sigma T^4 \quad \text{in terms of } T$$

Answers to Problems

Powers of ten

$$10^7 = 10,000,000$$

$$10^2 = 100$$

$$10^5 = 100,000$$

$$10^{-4} = 0.0001$$

$$10^{-6} = 0.000001$$

$$10^{-2} = 0.01$$

$$100,000 = 10^5$$

$$10 = 10^1$$

$$0.001 = 10^{-3}$$

$$0.0001 = 10^{-4}$$

$$10,000,000 = 10^7$$

Multiplying powers of 10

$$10^2 \times 10 = 10^3$$

⋮

$$10^6 \times 10^7 = 10^{13}$$

$$10^{-2} \times 10^4 = 10^2$$

$$10^{26} \times 10^3 = 10^{29}$$

$$10^{-34} \times 10^9 = 10^{-25}$$

$$10^4 \times 10^7 \times 10^6 = 10^{17}$$

$$10^2 \times 10^{26} \times 10^{-12} = 10^{16}$$

$$10^7 \times 10^{-4} \times 10^3 = 10^6$$

$$10^{12} \times 10^{-4} \times 10^3 = 10^{11}$$

$$10^{-34} \times 10^{24} \times 10^5 = 10^{-5}$$

Dividing by powers of 10

$$\frac{10^7 \times 10^9 \times 10^{-3}}{10^2} = 10^{11}$$

$$\frac{10^{-2} \times 10^4 \times 10^6}{10^{-3} \times 10^{-2}} = 10^{13}$$

$$\frac{1}{10^2 \times 10^5 \times 10^{-3}} = 10^{-4}$$

$$\frac{10^9}{10^2 \times 10^{-34} \times 10^{-27}} = 10^{68}$$

$$\frac{10^{16} \times 10^{-9} \times 10^4}{10^{-14}} = 10^{25}$$

Scientific notation

$$4 \times 100 \times 6.6 \times 10^{-2} = 26.4$$

$$7 \times 10 \times 3.14 \times 2 \times 10^3 = 4.396 \times 10^5$$

Powers in general

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^6 = 64$$

$$3^2 = 9$$

$$5^4 = 625$$

$$(10^4)^3 = (10^4) \times (10^4) \times (10^4) = 10^{(4 \times 3)} = 10^{12}$$

$$(10^5)^4 = (10^5) \times (10^5) \times (10^5) \times (10^5) \\ = 10^{(5 \times 4)} = 10^{20}$$

Square roots

$$\sqrt{49} = 7$$

$$\sqrt{36} = 6$$

$$\sqrt{20} = 4.4721$$

Logarithms

$$\log 10^4 = 4$$

$$\log 10^7 = 7$$

$$\log 10^5 = 5$$

$$\log 10^{-2} = -2$$

$$\log 10^{-8} = -8$$

$$\begin{aligned}\log 5 &= 0.6990 \\ \log(2010) &= 3.3032 \\ \log(0.45) &= -0.3468 \\ \log 75000 &= 4.8751\end{aligned}$$

$$\begin{aligned}\log(2 \times 10) &= \log 2 + \log 10 = 1.3010 \\ &= \log 20 = 1.3010 \\ &\text{(alternative method)}\end{aligned}$$

$$\log \frac{4}{2} = 0.3010$$

$$\log \frac{9}{3} = 0.4771$$

$$2.5 \log 14 = 2.8653$$

$$3.5 \log \frac{21}{3} = 2.9578$$

$$7 \log \frac{21 \times 2}{3} = 8.0229$$

Inverse logarithms

$$\log x = 4.5 \quad x = 31,623$$

$$\log x = 3.2 \quad x = 1,585$$

$$\log x = 6.5 \quad x = 3,162,278$$

$$\log x = -2 \quad x = 0.01$$

Order of evaluation

$$\begin{aligned}5 - 3 + 2.5 \log \frac{20}{5} &= 5 - 3 + 2.5 \times \log 4 \\ &= 2 + 2.5 \times 0.6021 \\ &= 3.5051\end{aligned}$$

$$\begin{aligned}6 + 4 \log \frac{10^5}{2} &= 6 + 4 \times \log(5 \times 10^4) \\ &= 6 + 4 \times 4.6990 \\ &= 24.796\end{aligned}$$

$$45 + \log 34 = 45 + 1.531 = 46.531$$

Trigonometric functions

$$\sin 45^\circ = 0.7071$$

$$\sin 90^\circ = 1.0$$

$$\sin 6^\circ = 0.1045$$

$$\sin 0^\circ = 0.0$$

$$\cos 0^\circ = 1.0$$

$$\tan 0^\circ = 0.0$$

$$\sin 90^\circ = 1.0$$

$$\cos 90^\circ = 0.0$$

$$\tan 45^\circ = 1.0$$

Inverse trig functions

BC	AC	Sin A	A
5	6	0.8333	56.44
2	5	0.4	23.58
1.5	4	0.375	22.02

Using symbols in equations

Area of a 10p...
Diameter = 24 mm

$$\begin{aligned}A &= \pi r^2 \\ &= \pi \left(\frac{24}{2} \right)^2 \\ &= \pi \times 144 \\ &= 452.39 \text{ mm}^2\end{aligned}$$

	m_2	b_1	b_2	Answer
1	6	65	40	5.473
2	4	350	12	0.338
3	5	137	4	1.163
4	8	14	30	8.827

Rearranging equations

$$y = mx + c$$

$$y - c = mx$$

$$m = \left(\frac{y - c}{x} \right)$$

$$L = 4\pi r^2 \sigma T^4$$

$$\frac{L}{4\pi \sigma r^2} = T^4$$

So

$$\begin{aligned}T &= \left(\frac{L}{4\pi \sigma r^2} \right)^{1/4} \\ &= \left(\frac{L}{4\pi \sigma} \right)^{1/4} \frac{1}{r^{1/2}}\end{aligned}$$